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Temperature Dependence of the Third-Order Elastic Constants of $SrTiO_3$

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The second-harmonic generation technique has been used to measure the combinations of third-order elastic constants C_{111}, $\frac{1}{4}(C_{111}+3C_{112}+12C_{166})$, and $\frac{1}{9}(C_{111}+6C_{112}+12C_{144}+24C_{166}+2C_{123}+16C_{456})$ relative to the room-temperature values over the temperature interval 106–300°K for strontium titanate single crystals. They are observed to exhibit even larger variations than the second-order constants. The observed decrease in C_{111} near the 102.5°K phase transition supports earlier observations that the crystal undergoes spontaneous strain in the [100] direction. Based on the assumption that the Cauchy relations extended to third-order elastic constants are valid above the phase transition, all six of the third-order constants are determined as a function of temperature.

I. INTRODUCTION

It is known that large changes in the mechanical properties of strontium titanate occur as a function of temperature near the 102.5°K phase transition. In particular, the work of Bell and Rupprecht[1] shows that a pronounced variation in the second-order elastic constants occurs and indicates that there is a large change in the ultrasonic attenuation. Cowley[2] has considered the lattice dynamics of strontium titanate and has argued that there is at least qualitative agreement between the temperature dependence of the elastic constants as measured by Bell and Rupprecht and that expected theoretically in the harmonic approximation. The present work was undertaken as a first attempt to determine the nonlinear elastic behavior of $SrTiO_3$ in the vicinity of the cubic-tetragonal phase change.

The experiment was to measure the third-order elastic (TOE) constants by use of the ultrasonic second-harmonic generation technique. The method was basically the same as that used by Gauster and Breazeale[3] at room temperature. An initially sinusoidal longitudinal wave of finite amplitude is introduced into the solid under study, and as the wave propagates, harmonics of the wave frequency are generated. The nonlinear equation of motion has been solved[4,5] for waves propagating along the principal axes of cubic crystals, and the amplitude of the second-harmonic component can be expressed as

$$A_2=-[(3K_2+K_3)/8K_2](A_1k_1)^2a, \tag{1}$$

where A_1 is the fundamental amplitude, k_1 is the fundamental wave vector, a is the distance of propagation from a sinusoidal driver, and K_2 and K_3 are the combinations of second- and third-order elastic constants given in Table I. If the second-order constants are known, the TOE constants can be calculated from wave-amplitude measurements.

When the elastic wave is reflected from a stress-free boundary, the fundamental and second-harmonic amplitudes which appear in Eq. (1) can be measured with a capacitive microphone. An equivalent circuit of the microphone is an ideal voltage source V in series with a capacitance equal to the capacitance of the parallel-plate microphone. Expressed in terms of the wave amplitude A_i (where $i=1$ or 2 for the fundamental or second harmonic, respectively), the bias voltage V_B, and the quiescent parallel-plate spacing S, the amplitude of the source voltage can be written as $V=2A_iV_B/S$. In the normal measurement of wave amplitudes, the voltage V is determined by replacing the microphone with a calibration circuit which has a signal generator in series with a capacitor equal to the microphone quiescent capacitance. Although the calibration scheme is quite satisfactory at a given temperature, it is very difficult to perform if calibration is required at many different temperatures.

Peters[6] has devised a method of relative measurements which allows one to measure the values of K_3 relative to a fixed temperature at which they are known. To describe the technique, we write Eq. (1) as

$$A(T)=K(T)k_1^2(T)a(T), \tag{2}$$

where

$$A(T)=A_2/A_1^2 \tag{3}$$

and

$$K(T)=-(3K_2+K_3)/8K_2. \tag{4}$$

Relative to a reference temperature T_0 $K(T)$ may be

TABLE I. SOE and TOE constant combinations for the [100], [110], and [111] directions in a cubic crystal.

Direction	K_2	K_3
[100]	C_{11}	C_{111}
[110]	$\frac{1}{2}(C_{11}+C_{12}+2C_{44})$	$\frac{1}{4}(C_{111}+3C_{112}+12C_{166})$
[111]	$\frac{1}{3}(C_{11}+2C_{12}+4C_{44})$	$\frac{1}{9}(C_{111}+6C_{112}+12C_{144}$ $+24C_{166}+2C_{123}+16C_{456})$

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[1] R. O. Bell and G. Rupprecht, Phys. Rev. **129**, 90 (1963).

[2] R. A. Cowley, Phys. Rev. **134**, A981 (1964).

[3] W. B. Gauster and M. A. Breazeale, Phys. Rev. **168**, 655 (1968).

[4] M. A. Breazeale and J. Ford, J. Appl. Phys. **36**, 3486 (1967).

[5] R. N. Thurston and M. J. Shapiro, J. Acoust. Soc. Am. **41**, 1112 (1967).

[6] R. D. Peters, Ph.D. dissertation, The University of Tennessee, 1968 (unpublished).

written as

$$K(T)=[k_1^2(T_0)a(T_0)/k_1^2(T)a(T)]B(T)K(T_0), \quad (5)$$

where

$$B(T)=A(T)/A(T_0). \quad (6)$$

If the quantities in Eq. (5) are known at the reference temperature T_0 and if $k_1^2(T)$, $a(T)$, and $A(T)$ are measured, then $K(T)$ can be determined. Of particular importance and convenience in the method, is that $B(T)$ can be expressed in terms of quantities that can be easily and accurately measured, and once $B(T)$ is known, $K_3(T)$ can be calculated by combining Eqs. (4) and (5). The fundamental elastic wave amplitude is adjusted to the same value at each temperature so that the factor $B(T)$ can be written as

$$B(T)=[S(T_0)/S(T)]V_{B2}(T_0)/V_{B2}(T), \quad (7)$$

where $S(T)$ is the parallel-plate microphone gap spacing at the temperature T, and $V_{B2}(T)$ is the microphone bias voltage required to keep the second-harmonic signal output equal to the output at the reference temperature. If $K(T_0)$ is known, then measurement of $K(T)$ reduces to the measurement of the wave velocity, sample length, gap spacing, and detector bias voltage. Combining Eqs. (4) and (5) and solving for $K_3(T)$, we have

$$K_3(T)=[3K_2(T_0)+K_3(T_0)] \times\left(\frac{K_2(T)}{K_2(T_0)}\right)^2 \frac{\rho(T_0)a(T_0)}{\rho(T)a(T)}B(T)-3K_2(T), \quad (8)$$

where ρ is the sample density, and $B(T)$ is defined by Eq. (6).

Calibration errors have limited measurements of $K_3(T_0)$ to approximately ±8.5%, but relative measurements improve the precision such that measurements can be reproduced to within ±1%. Values of $K_3(T_0)$ have been determined for SrTio$_3$ by Mackey and Arnold,[7] and the values of $K_2(T)$ have been reported by Bell and Rupprecht.[1]

In the present work, values of $K_3(T)$ have been calculated from the measured values of $B(T)$ substituted into Eq. (8). As expected from the behavior of the second-order constants near the 102.5°K phase change the linear combination of TOE constants changes rapidly near the transition temperature. The experimental error reported for the values of $K_3(T_0)$, where $T_0=300°K$ was ±8.6%, and combining the small errors involved in the relative measurements, we report an error in the $K_3(T)$ of ±10%.

II. MEASUREMENTS

A. Experimental System

A block diagram of the experimental system is shown in Fig. 1. An Arenberg pulser was operated in the gated amplifier mode, and the frequency of the pulser was controlled by a cw rf oscillator. Frequency was monitored continuously by a frequency counter and scaler combination and was held constant to within four parts in 10^5. The pulsed rf output of the pulser was applied to an X-cut quartz driving transducer, and 30-MHz

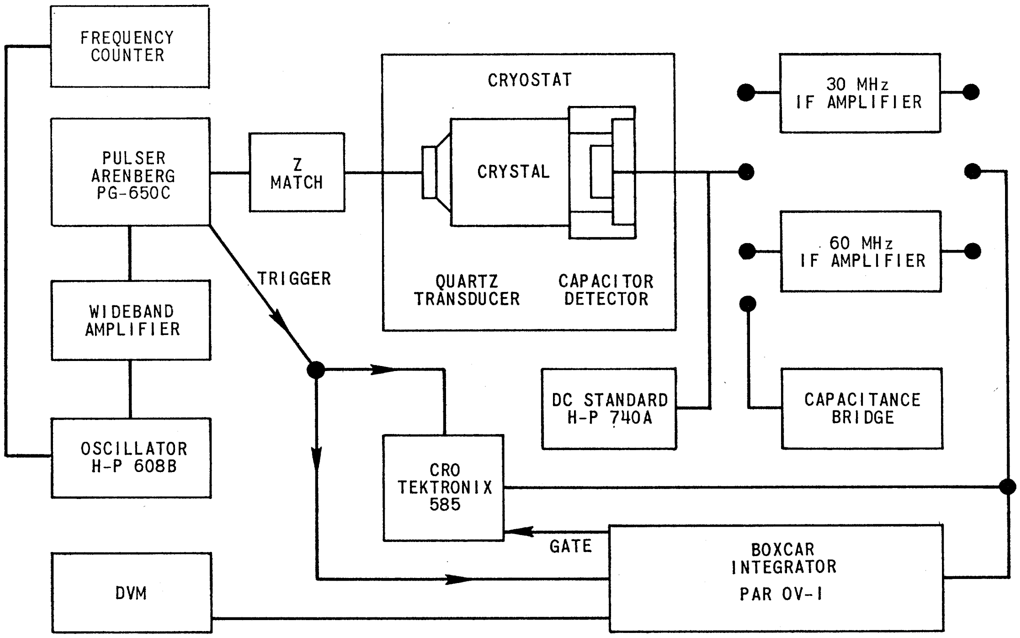

FIG. 1. Experimental system.

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[7] James E. Mackey and R. T. Arnold, J. Appl. Phys. 40, 4806 (1969).

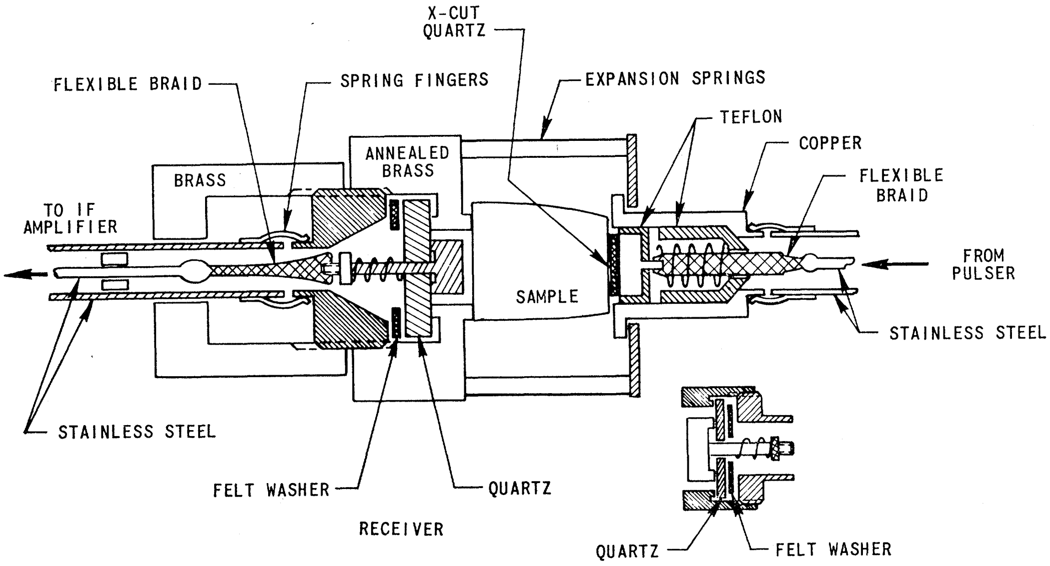

FIG. 2. Quartz transducer driver, the capacitor driver (shown in the insert), and the capacitor microphone.

pulses of about 4-μsec duration and approximately 300-V peak amplitude excited the transducer. Samples under study were in the form of boules with two opposing faces perpendicular to the boule axis. The X-cut quartz was bonded to one face, and the opposing surface (coated with 2000 Å of evaporated Al) served as one plate of a parallel-plate capacitor microphone which was biased from a dc standard voltage supply. Motion of the end of the sample generated a signal which was selectively amplified and rectified by either a 30 or a 60-MHz i.f. amplifier.[8] The rectified pulses from the i.f. amplifier and the gate signal from the Princeton Applied Research Boxcar Integrator were observed simultaneously on an oscilloscope. Any pulse in the echo pattern could be selected for amplitude measurement by setting the gate signal of the boxcar coincident with the echo pulse. A digital voltmeter was used to read the output voltage of the boxcar which was proportional to the sound-wave amplitude.

An important experimental consideration in the relative amplitude measurements was that of standing waves in the coaxial line connecting the microphone to the i.f. amplifiers. To minimize errors resulting from changes in standing-wave ratio (SWR) with temperature, the impedance of the line was matched to the i.f. amplifier (93 Ω), the fundamental frequency was held constant (4: 10^5), and the microphone and coaxial line were designed such that the electrical characteristics did not vary markedly with temperature. The coaxial line inside of the cryostat was made of stainless-steel conductors and an air dielectric. Over the entire interval from 100–300°K the maximum change in line capacitance was 0.2 pF. The microphone was constructed from annealed brass, and the capacitance was observed to change by ±1% over the entire temperature interval. (In the previous application of the relative technique by Peters, a constant capacitance microphone was used which had an adjustable gap spacing.[6,9]

Figure 2 is a diagram of the capacitive driver, transducer driver, capacitor microphone, sample, and sample holder. The capacitor microphones were both made of free-turning yellow brass, and the capacitive transducers were prepared such that the gap spacing was approximately 5 μ. The microphone and driver plates were $\frac{3}{8}$ and $\frac{1}{2}$ in. in diam, respectively, and the plate surfaces were flat and parallel to within 0.25 μ. After placing the sample holder inside of a sealed Dewar, the Dewar was completely submerged in liquid nitrogen. A fan driven by a motor located outside of the cryostat was used to continuously circulate helium gas inside the inner Dewar in order to insure rapid approach to thermal equilibrium. A silicon-controlled rectifier proportioning controller with a platinum-resistance sensor was used to control the temperature, and a copper constantan thermocouple with an ice bath reference and a Rubicon potentiometer was used to measure the temperature. Changes of less than 0.025°K could be detected and the temperature of the sample could be brought to a control point and maintained for hours with no detectable temperature change. The error in temperature measurements was estimated to be ±1°K, and by comparison of the temperature measured for the phase transition in the present work

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[8] It is interesting to note that a small signal could be observed from the microphone when the applied bias voltage was zero. The signal was observed only when the capacitive microphone plates were made of different metals. A similar observation was reported by W. Zisman as a method of measuring contact potential difference [Rev. Sci. Instr. **3**, 367 (1932)].

[9] R. D. Peters, M. A. Breazeale, and V. K. Pare, Rev. Sci. Instr. (to be published).

(103°K) with that of Rupprecht and Winter (102.5 ±0.1°K), it was felt unnecessary to devise a more accurate system.

B. Temperature Dependence of Second-Order Elastic Constants

The temperature dependence of the second-order elastic (SOE) constants of $SrTiO_3$ has been measured by Bell and Rupprecht[1] between room temperature and the temperature at which the cubic-tetragonal phase change occurs, and a value of 108°K was reported for the transition temperature T_a. More recently, Rupprecht and Winter[10] have presented a more accurate value of $T_a=(102.5\pm0.1)$°K. Since the elastic properties change very rapidly near the phase transition, an experiment was conducted to check the temperature dependence of the SOE constant C_{11} reported by Bell and Rupprecht.

Use of the double-pulse method of Williams and Lamb[11] allowed the wave velocity to be measured within ±0.1%. By using a capacitive microphone as both a driver transducer and as a receiving microphone,[12] both ends of the sample were free, and it was unnecessary to make phase corrections for the reflected waves. The experimental system used to measure the wave velocity was the same as that shown in Figs. 1 and 2 except that a parallel-plate capacitive transducer was used instead of the 30-MHz X-cut quartz driver (see the insert in Fig. 2).

C. Temperature Dependence of Attenuation

In the theoretical results of Breazeale and Ford[4] and Thurston and Shapiro,[5] it was assumed that attenuation can be neglected. Gauster and Breazeale[3] have reported an approximation for the affect of attenuation on the wave-amplitude measurements where they have assumed that the attenuation coefficients for the fundamental and second harmonic can be measured independently. In this approximation, Eq. (1) becomes

$$A_2=\frac{3K_2+K_3}{8K_2}(A_1k_1)^2a\frac{e^{-2\alpha_1a}-e^{-\alpha_2a}}{(\alpha_2-2\alpha_1)a}, \qquad (9)$$

where α_1 and α_2 are the fundamental and second-harmonic attenuation coefficients, respectively.

A method similar to that reported by Arnold *et al.*[12] was used to determine α_1 and α_2 as a function of temperature. The experimental system was identical to that used for the wave velocity measurements, and the measurements were performed similar to the relative-amplitude technique used for the measurement of $K(T)$. To measure the attenuation at a given temperature the boxcar gate signal was used to pick out the first pulse in the echo pattern, and the bias on the microphone was adjusted to a value ${}_1V_B$ such that the output of the boxcar was V. The gate signal was then delayed in order to pick out the second pulse in the echo pattern. By increasing the microphone bias to a new level of ${}_2V_B$, the boxcar output could be maintained at the voltage V. The attenuation coefficient could then be calculated from

$$\alpha=(1/2a)\ln({}_2V_B/{}_1V_B), \qquad (10)$$

where a is the sample length.

D. Sample Preparation and Transducer Bond

The strontium titanate crystals used in this experiment were grown in the form of boules by the Titanium Division of the National Lead Co. One end of each crystal was cut by the supplier so that the surface was perpendicular to within 3° of either the [100], [110], or [111] axis. A diamond saw was used to cut the crystals to the desired length, and both ends were ground and polished flat to within 0.25 μ and parallel to within 12 sec of arc. Aluminum surfaces were evaporated on the parallel faces in order to provide electrical grounding to both the driver and receiver assemblies.

The quartz transducers were bonded to the sample with Leech Products, Inc., X-30 contact cement which was chosen because it is a good general purpose glue at liquid-nitrogen temperatures. While the transducers were being bonded to the samples, a monochromatic light source was used to observe the interference patterns in the bond. The cement was carefully pressed out until the bond was 2–3 μ thick, and the surfaces of the transducer and sample were parallel to within 0.5 μ. Acoustical properties of the bonds prepared in this manner were very reproducible after drying for a period of 24 h.

III. RESULTS

A. Temperature Dependence of $B(T)$

Figures 3–5 show the temperature dependence of the factor $B(T)$ defined by Eq. (6) for $SrTiO_3$ crystals

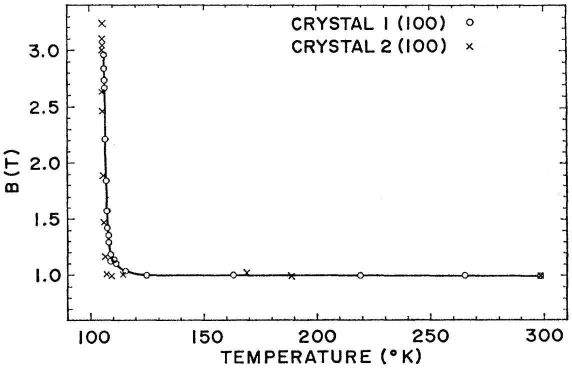

FIG. 3. Factor $B(T)$ for the [100] direction. Data are presented for two different crystals.

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[10] G. Rupprecht and W. H. Winter, Phys. Rev. **155**, 1019 (1967).

[11] J. Williams and J. Lamb, J. Acoust. Soc. Am. **30**, 308 (1958).

[12] R. T. Arnold, James E. Mackey, and E. L. Meeks, J. Acoust. Soc. Am. **42**, 677 (1967).

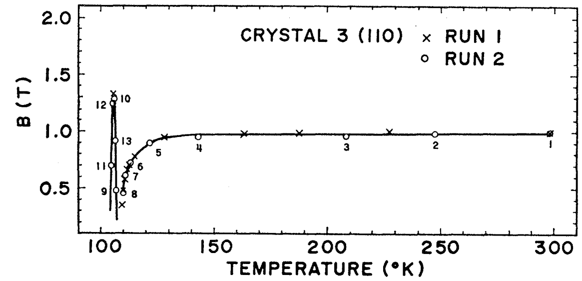

FIG. 4. Factor $B(T)$ for the [110] direction. The numbered points indicate the sequence in which the data were recorded as the sample was cooled below 100°K and then warmed again for the data of run 1 (circles). The crosses (run 1) represents a separate measurement on the same crystal.

oriented in the [100], [110], and [111] directions, respectively. Figure 3 shows data obtained on two [100] crystals, and the data for the two crystals exhibit the same temperature dependence over the interval 115–300°K. Although the same rapid change in $B(T)$ was observed near 103°K, below 115°K the values of $B(T)$ for the two crystals differed by more than expected on the basis of the experimental error. We have interpreted the difference as real and are of the opinion that it occurs because sound propagation is off axis by a different amount in each of the two crystals.

Data for two completely separate measurements on the same [100] oriented crystal are presented in Fig. 4. Between the two sets of data, the quartz transducer and capacitive microphone were removed from the sample, and the microphone was completely reassembled. The data demonstrate that the reproducibility of the measurements was very good.

In all of the measurements of $B(T)$, data were recorded for increasing and decreasing temperatures as the crystal was allowed to go through the phase transition. No thermal hysterisis was observed within the precision of the temperature meaasurement, ±0.025°K.

B. Temperature Dependence of SOE Constants

In their work, Bell and Rupprecht[1] have presented the analytical expressions for $C_{11}(T)$, $C_{12}(T)$, and $C_{44}(T)$ which are reproduced in Table II. Figure 6 presents data of the present work for $C_{11}(T)$ (open circles) and the values of $C_{11}(T)$ calculated from the expression in Table II with a transition temperature T_a of 102.5°K. The value of $T_a = (102.5 \pm 0.1)$°K is that reported by Rupprecht and Winter.[10] On the basis of the very good agreement between our measurements of C_{11} and the values calculated from the expression given by Bell and Rupprecht, we have concluded that the values of $C_{11}(T)$, $C_{12}(T)$, and $C_{44}(T)$ could be calculated from expressions (1), (2), and (3) in Table II.

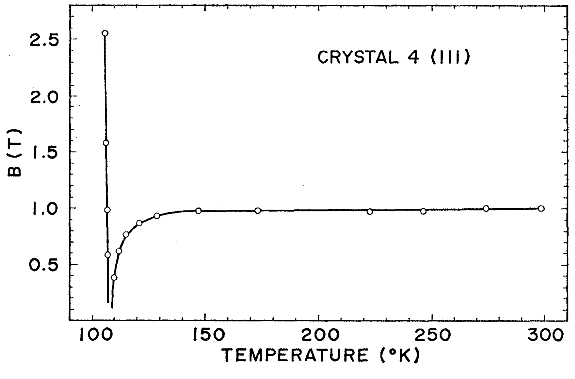

FIG. 5. Factor $B(T)$ for the [111] direction.

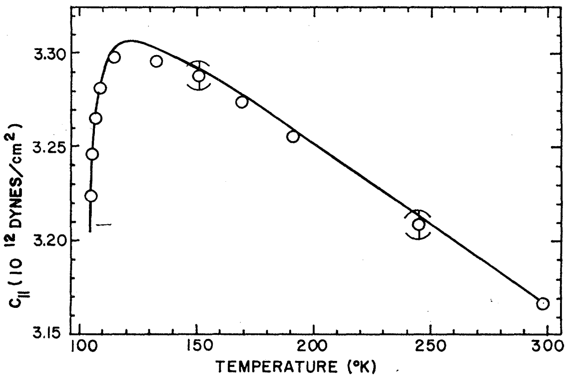

FIG. 6. Temperature dependence of C_{11} for strontium titanate. The solid line was determined from the equation reported by Bell and Rupprecht (Ref. 1) using $T_a = 102.5$°K.

C. Temperature Dependence of Attenuation Coefficients

Figure 7 presents the data obtained by use of the double microphone technique for the temperature dependence of the 30 and 60 MHz attenuation coefficients for a [100] oriented crystal. The estimated error is ±10%. The attenuation coefficients are very small over the interval from 107–300°K. Calculated values for the diffraction loss were determined from the results of Papadakis,[13] and are also presented in Fig. 7 (solid lines). Below 107°K the attenuation was observed to increase very rapidly, and an echo pattern could not be observed below 103°K.

Attenuation data for the [110] and [111] crystals were not recorded at any temperature except 300°K. Comparison of the echo patterns during the harmonic generation measurements for the three crystal orientations indicated that there was no observable difference in the attenuation coefficients for the three crystal

TABLE II. Temperature dependence of the SOE constants (Ref. 1) of $SrTiO_3$.

(1) $C_{11}(T) = 3.341 \times 10^{12}[1 - 2.62 \times 10^{-4}(T - T_a) - 0.0992/(T - T_a)]$
(2) $C_{12}(T) = 1.049 \times 10^{12}[1 - 1.23 \times 10^{-4}(T - T_a) + 0.1064/(T - T_a)]$
(3) $C_{44}(T) = 1.267 \times 10^{12}[1 - 1.30 \times 10^{-4}(T - T_a) - 0.1242/(T - T_a)]$

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[13] E. P. Papadakis, J. Acoust. Soc. Am. **40**, 863 (1966).

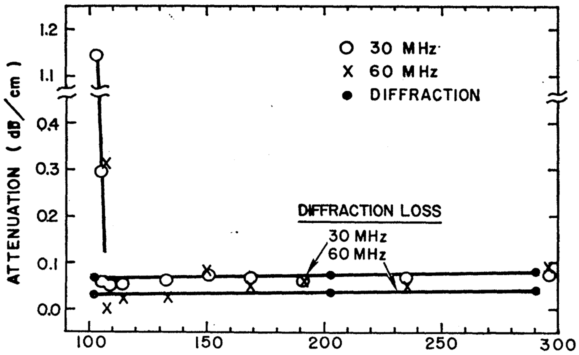

FIG. 7. Attenuation of 30- and 60-MHz longitudinal waves in the [100] direction of strontium titanate. The solid lines represent the calculated diffraction losses (Ref. 13).

orientations. In all observations the attenuation above 107°K was no larger than the room-temperature values.

IV. DISCUSSION

The TOE constant combinations K_3 presented in Table I were calculated from Eq. (8) for temperatures greater than 107°K, and the results are presented in Figs. 8–10. Data used in the calculations were as follows: (a) the values of $B(T)$ shown in Figs. 3–5; (b) the SOE constants calculated from the expressions listed in Table II; and (c) the room-temperature values of K_3 listed in Table III. For temperatures above 107°K, corrections for both attenuation and thermal expansion were very small and were not made. Below 107°K, interpretation of the data for K_3 is very difficult. The data indicate that there is a change of sign in both $K_3[110]$ and $K_3[111]$ just below 107°K. Although the phase of the second-harmonic signal relative to the fundamental signal was not measured in the experiment, when it was assumed that a 180° phase change in the harmonic occurred near 107°K, some of the data recorded below 107°K fit the empirical expressions for K_3 given in Table IV. An experimental value of $K_3[110]$ at a temperature of 106°K was calculated on this basis and is indicated by the triangular point plotted in Fig. 9. The sound echo pattern for the [110] oriented crystal was observed to exhibit a very complicated behavior between 103–108°K, and the pattern actually

TABLE III. Room-temperature values of the TOE constant combinations K_3 for strontium titanate.[a]

Direction	K_3
[100]	$-49.6\pm4.3\times10^{12}$ dyn/cm²
[110]	$-27.3\pm3.0\times10^{12}$ dyn/cm²
[111]	$-27.9\pm2.6\times10^{12}$ dyn/cm²

[a] J. E. Mackey and R. T. Arnold (see Ref. 7).

TABLE IV. Temperature dependence of the TOE constant combinations K_3.

Directon	$K_3(T)$
[100]	$K_3(T)=-5.84\times10^{13}(\exp\{[2.49/(T-T_a)]^{3.5}\}-5.07\times10^{-4}T)$
[110]	$K_3(T)=-2.98\times10^{13}(\exp\{-[6.10/(T-T_a)]^{2.3}\}-1.57\times10^{-4}T)$
[111]	$K_3(T)=-2.89\times10^{13}(\exp\{-[4.74/(T-T_a)]^{2}\}-1.38\times10^{-4}T)$

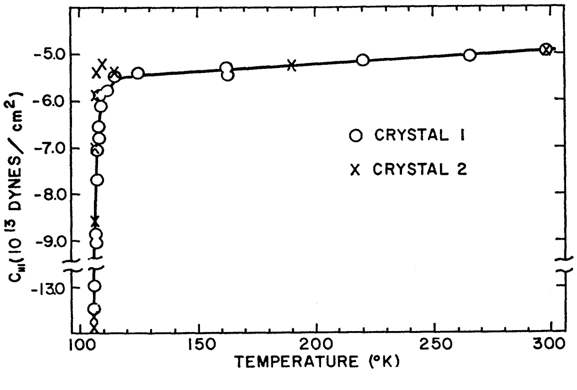

FIG. 8. Temperature dependence of C_{111} for strontium titanate. Data are presented for two different crystals. The solid line was determined from the empirical relation for [100] listed in Table IV.

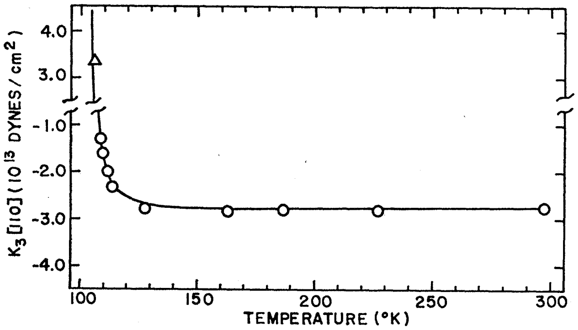

FIG. 9. Temperature dependence of the TOE constant combinations for the [110] direction. The solid line was determined from the empirical relation for [110] listed in Table IV, and the triangular point was calculated from the experimental data by assuming $(K_2+3K_3)/8K_2$ is a positive quantity.

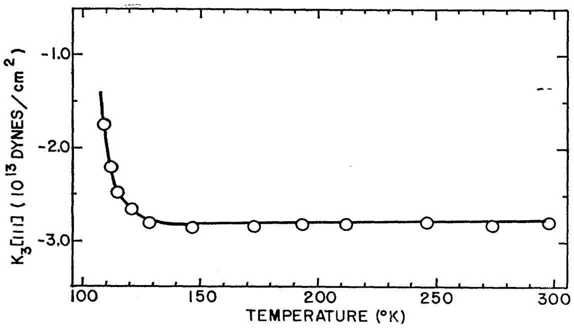

FIG. 10. Temperature dependence of the TOE constant combinations for the [111] direction. The solid line was determined from the empirical relation for [111] listed in Table IV.

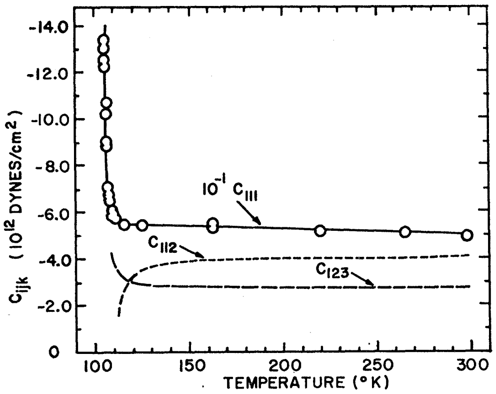

FIG. 11. Temperature dependence of the TOE constants for strontium titanate determined on the basis that the extended Cauchy relations hold above the 102.5°K phase transition.

disappeared at 108°K and reappeared in the narrow interval between 104–107°K before completely disappearing at 104°K. A detailed evaluation of the data below 108°K is likely to be of little value until more is known about the sound attenuation.

The only TOE constant that can be determined directly from the data is C_{111}. If the Cauchy relations extended to third-order constants ($C_{112}=C_{166}$ and $C_{144}=C_{123}=C_{456}$) are assumed to hold above the transition temperature, then all six of the TOE constants can be calculated. Figure 11 shows the temperature dependence of the TOE constants for $SrTiO_3$ determined on the basis that the extended Cauchy relations are valid, and the empirical relations listed in Tables II and IV were used in the calculation. A possible justification for this assumption for $SrTiO_3$ is that the Cauchy relation ($C_{12}=C_{44}$) for the SOE constants is a reasonable approximation at room temperature and becomes even a better approximation near the phase transition.[1] Although the validity of the assumption for the TOE constants is certainly questionable, the results shown in Fig. 11 suggest a rapid change in the TOE constants near the cubic-tetragonal phase change.

It is likely that a theoretical calculation of the TOE constants for strontium titanate will at best be very tedious. Although several authors have calculated the constants for the alklai halides[14,15] and certain cubic metals,[16] it is not clear exactly how one should proceed for the perovskite structure. The theoretical results thus far are in reasonable agreement with existing experimental data for both the alkali halides and cubic metals. In both cases, the short-range repulsive force is found to be quite predominant over other forces in the calculated TOE constants. It is interesting to note that the experimental values of K_3/K_2 for strontium titanate above 170°K agree reasonably well with values calculated by Ghate[14] for the NaCl-type alkali-halide crystals. A similar comparison with the theoretical values for the CsCl-type halides and the cubic metals yields a much poorer agreement. Comparison of the temperature variation of the K_3's indicate no direct correlation between strontium titanate and either alkali-halide structure.

The measured temperature dependence of C_{111} supports the idea that the restoring force for [100] displacements tends to zero at the transition temperature, and the crystal undergoes a spontaneous strain in the [100] direction at T_a. Further, the observed change in attenuation near T_a supports the idea that domains are formed as a consequence of the spontaneous deformation.

The measured temperature dependence of the 30-MHz attenuation coefficient agrees well with the calculated diffraction loss. This has been interpreted to indicate that either the dislocation density is low or the dislocations are pinned. The 60-MHz attenuation coefficient was observed to exhibit some temperature dependence greater than expected from the diffraction loss calculations. A small electrical pickup signal was observed to occur coincident with the applied rf during the 60-MHz measurements. We are not certain that the trailing edge of this pulse did not overlap the first reflected pulse to the extent that the measured amplitude of the reflected pulse was changed.

Work is in progress to determine the six TOE constants for $SrTiO_3$ by the method of uniaxial stress, to investigate the attenuation in the vicinity of the cubic-tetragonal phase transition, and to try to calculate the TOE constants using a Born-Mayer potential for the short-range repulsive force.

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[14] P. B. Ghate, Phys. Rev. **139**, A1666 (1965).

[15] A. A. Nran'yan, Fiz. Tverd. Tela **5**, 177 (1963); **5**, 1865 (1963) [English transls.: Soviet Phys.—Solid State **5**, 129 (1963); **5**, 1361 (1964)].

[16] R. C. Lincoln, K. M. Koliwad, and P. B. Ghate, Phys. Rev. **157**, 463 (1966).